

**WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH**

**PROBLEM SET V (2025–2026)**

**February 2026**

1. Do there exist positive integers  $a$  and  $b$  such that both  $a^2+4b$  and  $b^2+4a$  are perfect squares?
2. Parallelogram  $ABCD$  has integer side lengths with  $AB = 8$ . The bisectors at  $A$  and  $B$  intersect at a point  $P$  which is outside of the parallelogram. What can be the length of  $BC$ ?
3. A teacher has a bag of 100 balls numbered from 1 to 100. She picks one ball at random, writes its number on the board, and puts it back in the bag. 100 students line up in alphabetical order, and each student takes a turn doing the same (pick a ball randomly, write its number on the board, return the ball to the bag). The first student to draw a ball whose number is already on the board wins a prize. Which student has the best chance of winning?
4. Alice and Bob are playing a game. At the beginning of the game, there are 2026 piles, consisting of 1, 2, 3, . . . , 2025, and 2026 blocks, respectively. The two players take turns, starting with Alice. Each player finds the pile with the most blocks (or one of such piles, if there are several) and proceeds to take any nonzero number of blocks from it. The player who takes the last block wins. Which player has a winning strategy?
5. The captain of a pirate ship wants to distribute pieces of a gold bar among his three crew mates. He first picks a number  $q$  with  $0 < q < 1$ , cuts off a piece that is a  $q$  proportion of the gold bar, and gives that piece to one of the three crew members. At every next step, the captain cuts a piece that is a  $q$  portion of the gold bar remaining from the previous step, and gives that piece to one of the members. (for example: if  $q = 0.234$ , then the captain always cuts off 23.4% of the remaining gold bar.) After a certain number of steps, the captain stops cutting pieces and saves the remaining gold bar for himself. Is it possible that at this point each of his three crew mates has received the same amount of gold?

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

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Find an introduction to techniques for solving problems like these at <https://go.wisc.edu/551pe6>

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<b>Deadline</b> March 15, 2026	
<b>Problem</b>	<b>Score</b>
1	
2	
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