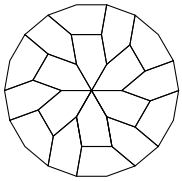
WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET III (2025–2026) **December 2025**

1. Jon claims that he can cut a regular 18-gon into eighteen identical pentagons (5-gons) and produced the picture on the right to justify his claim. Unfortunately, his picture is not for scale. Prove or disprove that the proposed arrangement of 18 identical pentagons within the regular 18-gon is possible.

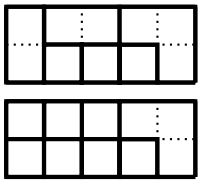


2. Suppose that $a \geq 2$ and $b \geq 1$ are integers. Show that $a^b - 1$ is divisible by $a - 1$, and that $(a^b - 1)(a^{b+1} - 1)(a^{b+2} - 1)$ is divisible by $(a - 1)(a^2 - 1)(a^3 - 1)$.

3. A group of mathematicians make 2025 statements, one after another. The first statement is unknown, but every statement starting from the second is identical: "Among the previous statements more than 70% are false." How many false statements will be made?

4. There are 150 students in a calculus class, each with exactly one first name and one last name. Each student writes on the board how many other students in the class have the same first name as themselves. Then each student writes on the board how many other students in the class have the same last name as themselves. When they are finished, among the numbers on the board, each of the numbers $0, 1, 2, \ldots, 23$ appears at least once. Show that at least two students in the class have the same first and last name.

5. Let a_n be the number of ways to place 1×1 tiles, 2×1 tiles, and 3-cell L shaped tiles to tile a $2 \times n$ grid of cells. (We do not need to use all three types of tiles.) For example, the picture shows two different ways to tile a 2×5 grid. Show that for every positive integer n, that $4^n \leq 2a_n \leq 5^n$.



You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

Find old and current problems and information about the talent search at: `http://www.math.wisc.edu/talent`

Find an introduction to techniques for solving problems like these at `https://go.wisc.edu/551pe6`

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