WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET V (2024–2025) **February 2025**

1. We have a rectangular table with at least two rows and at least two columns that contains 200 cells. We place one number in each cell so that the numbers in the first two rows and each of the columns form arithmetic progressions. The sum of all the numbers in the table is 101250. What is the sum of the four numbers in the corners of the table?

2. Write a positive integer $M > 1$ as the product of its prime factors. Then increase each of those prime factors by 1, and let N be the product of these new factors. Suppose that N is a multiple of M. Now write N as a product of its prime factors and increase each of those factors by 1. Prove that the product of these new factors will be a perfect square.

3. Let $ABCDEFGHIJK$ be a regular 11-gon (a polygon with 11 sides) with unit side length. Let M be the intersection point of the diagonals $\overline{AE}$ and $\overline{CF}$. Show that $AF = AM + 1$.

4. We have 2025 nonzero real numbers $a_1, a_2, \ldots, a_{2025}$ that are not all equal to each other. The numbers satisfy the following equations:

$$a_1 + \frac{1}{a_2} = a_2 + \frac{1}{a_3} = \cdots = a_{2024} + \frac{1}{a_{2025}} = a_{2025} + \frac{1}{a_1}.$$

   Show that the product of our 2025 numbers cannot be equal to 5.

5. On an 8×8 chessboard, someone placed coins in 35 of its 64 squares. (There is at most one coin in a square.) The Greedy Gnome is traveling from the lower left square to the upper right, moving only up or right on each step, collecting coins along the way. Show that the Gnome can always find an admissible path along which he can collect at least 6 coins, but we can position 34 coins on the board in a way that he will not be able to get more than 5.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

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