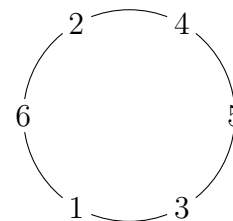


WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH
PROBLEM SET IV (2024–2025)

January 2025

- Find all positive prime numbers p for which $8p + 1$ is a perfect square.
- Consider a square $ABCD$. Let P be a point on the side \overline{BC} and Q a point on the side \overline{CD} such that $\angle BAP = \angle PAQ$. Show that $AQ = BP + QD$.
- For integer $n > 3$, the numbers $1, 2, 3, \dots, n$ are written in a circle in some order. Call a pair of these numbers *extraordinary* if (1) they are not next to each other on the circle, and (2) for one of the two arcs between them, all the numbers on that arc are less than these two numbers. (In the picture, $n = 6$, and the extraordinary pairs are $(4, 6)$, $(3, 6)$ and $(5, 6)$.) Show that no matter what order the numbers are written, there are always exactly $n - 3$ extraordinary pairs.
- A frog is hopping along the number line. Before each hop, we specify the distance the frog must travel, but the frog chooses the direction (either left or right). For example, if the frog is at position x and we specify a hop of $\frac{2}{3}$, the frog can move to either $x + \frac{2}{3}$ or $x - \frac{2}{3}$. The objective is to force the frog to eventually land on either position 0 or position 1. From which starting positions is it always possible to accomplish this goal?
- An analyst wants to study the network of airline routes in the US. For each airport, she records the number of other US airports that can be reached directly by a flight from it – this is the airport’s *connection number*. For each airport, she also computes the average of the connection numbers for those airports that can be reached by a direct flight from it – this is the airport’s *neighbor connection number*. Show that the average of the connection numbers is no greater than the average of the neighbor connection numbers. (Assume that if two airports are connected by a direct flight, then there are direct flights in both directions.)



You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

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