WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET III (2024–2025) **December 2024**

1. Four nonzero real numbers a, b, c, d satisfy the equation

$$\frac{a}{b}+\frac{b}{a}=\frac{c}{d}+\frac{d}{c}.$$

Show that the product of two of the numbers from a, b, c, d is equal to the product of the other two.

2. Given a regular 2025-gon, how many ways can we choose four of its 2025 vertices so that they are the four vertices of a trapezoid?

3. A small town has several bus lines. Each line has 3 stops and any two lines share no more than 1 stop. What is the maximum possible number of bus lines if there are 9 bus stops?

4. The convex pentagon $ABCDE$ has equal length sides, but all of its angles are different. Show that the largest and the smallest of its angles share the same side of the pentagon.

5. Show that the following expression is less than 3:

$$\sqrt{1+\sqrt{2+\sqrt{2^2+\sqrt{2^3+\sqrt{\cdots\sqrt{2^{2024}+\sqrt{2^{2025}}}}}}}}.$$

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

Find old and current problems and information about the talent search at: `http://www.math.wisc.edu/talent`
Find an introduction to techniques for solving problems like these at `https://go.wisc.edu/551pe6`

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Deadline January 11, 2025	
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