

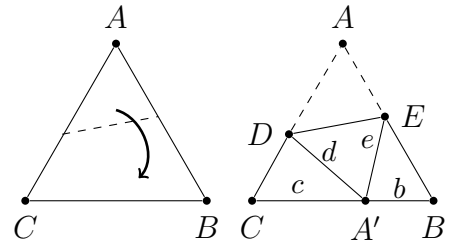
WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET II (2024–2025)

November 2024

1. We have 1000 white unit cubes. We build a $10 \times 10 \times 10$ cube from these, and then paint some (or all) of the faces of this bigger cube purple. After disassembling the bigger cube we found that there are an odd number of unit cubes with at least one purple face. How many faces of the $10 \times 10 \times 10$ cube were painted purple?
2. The sum of 7 distinct positive integers is at least 100. Show that we can choose 3 of these integers so that their sum is at least 50.

3. An equilateral triangle $\triangle ABC$ is made out of paper. Fold the paper, as shown on the right, so that vertex A ends up at a point A' on side \overline{BC} , and the fold line intersects sides \overline{AB} and \overline{AC} at points D and E , respectively. Let $b = A'B$, $c = A'C$, $d = A'D$, and $e = A'E$. Show that $\frac{d}{e} = \frac{b+2c}{2b+c}$.



4. Let $n = 2^{2024} \cdot 3^{2025}$. How many positive divisors does n^2 have that are less than n , but do not divide n ?
5. Ten boxes with 1, 2, 3, ..., 9, and 10 marbles are set on the table. Two players are taking turns picking one marble from any one of the boxes. The game ends when there are 3 marbles left. If the 3 marbles are in 3 different boxes, then the second player wins. Otherwise, the first player wins. Does one of the players have a winning strategy?

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

Find old and current problems and information about the talent search at: <http://www.math.wisc.edu/talent>

Find an introduction to techniques for solving problems like these at <https://go.wisc.edu/551pe6>

Return To	MATHEMATICS TALENT SEARCH Dept. of Mathematics, 480 Lincoln Drive University of Wisconsin, Madison, WI 53706	Deadline December 11, 2024	
Or Email To	talent@math.wisc.edu	Problem	Score
Please Fill In	PROBLEM SET II	1	
Name & Grade		2	
School & Town		3	
Home Address		4	
Town & Zip		5	
Email Address			
Teacher's Name			
Teacher's Email			