

WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET I (2024–2025)

October 2024

1. The sum of three integers is equal to 1, and their product is equal to 24. What can these three integers be?
2. Point  $M$  located inside  $\triangle ABC$  moves parallel to the side  $\overline{BC}$  until it intersects with the side  $\overline{CA}$ , then parallel to  $\overline{AB}$  until it intersects with  $\overline{BC}$ , then parallel to  $\overline{AC}$  until it intersects with  $\overline{AB}$ , and so forth. Prove that, after some number of steps, the trajectory of the point will be closed.
3. We draw a  $2024 \times 2024$  grid of unit squares. We call the vertices of the unit squares in the grid *lattice points*; there are  $2025^2$  of these in our grid. Someone chose 10 of these lattice points, and drew all the line segments that connect any two of them. Show that at least one of these drawn line segments will contain at least two more lattice points besides its end points.

4. Let  $a, b, c$  be positive real numbers such that  $abc = 1$ . Show that if

$$a + b + c > \frac{1}{a} + \frac{1}{b} + \frac{1}{c},$$

then exactly one of the three numbers is greater than 1.

5. Let  $n > 1$  be an integer. We wrote the numbers  $1, 2, 2^2, \dots, 2^{n-1}$  on a board, and circled the first digit of each number. Show that out of the nine possible digits  $1, 2, \dots, 9$  there is at least one that is circled at most  $\frac{n}{17}$  times.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

Find old and current problems and information about the talent search at: <http://www.math.wisc.edu/talent>

Find an introduction to techniques for solving problems like these at <https://go.wisc.edu/551pe6>

<b>Return To</b>	<b>MATHEMATICS TALENT SEARCH</b> Dept. of Mathematics, 480 Lincoln Drive University of Wisconsin, Madison, WI 53706	<b>Deadline</b> November 11, 2024	
<b>Or Email To</b>	talent@math.wisc.edu		
<b>Please Fill In</b>	<b>PROBLEM SET I</b>	<b>Problem</b>	<b>Score</b>
Name & Grade		1	
School & Town		2	
Home Address		3	
Town & Zip		4	
Email Address		5	
Teacher's Name			
Teacher's Email			