WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET V (2023-2024) **February 2024**

1. Initially, each entry in a 4×4 table is equal to 0. In each step we can choose a 2×2 subtable and increase each of the four numbers in that subtable by one. Decide if the following table can be obtained in a finite number of steps.

3	4	3	2
6	11	11	4
4	12	11	5
1	3	5	3

2. Maya wrote down a power of two on the board. Ian wrote down a different number by rearranging the digits of Maya's number. Show that Ian's number cannot be a power of two. (Ian cannot move a zero digit to the first position.)

3. We have a geometric progression of n positive integers with $n \geq 2$. (This means that the ratio of each two consecutive integers in the progression is the same.) Show that the average of all the n terms in the progression cannot be greater than the average of the first and last term of the progression.

4. There are 2024 triangles in the plane so that any two of them intersect with each other. Show that we can draw a straight line that intersects all the triangles.

5. A baker baked a rectangular pie and cut it into n^2 rectangles by making $n-1$ vertical cuts and $n-1$ horizontal cuts. (n is at least 2.) The areas of the resulting pie pieces rounded to the nearest integers are equal to all the natural numbers from 1 to n^2 in some order. What is the greatest n for which this is possible? (Semi-integer numbers are rounded upward.)

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

Find old and current problems and information about the talent search at: `http://www.math.wisc.edu/talent`

Find an introduction to techniques for solving problems like these at `https://go.wisc.edu/551pe6`

Return To **Or Email To**	**MATHEMATICS TALENT SEARCH** Dept. of Mathematics, 480 Lincoln Drive University of Wisconsin, Madison, WI 53706 talent@math.wisc.edu	**Deadline** March 12, 2024

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Problem	**Score**
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