**WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH**

**PROBLEM SET III (2023-2024)** **December 2023**

1. How many positive integers can you find so that the 3rd largest divisor of the number is equal to 2024? (We count the number itself as a divisor.)
2. An arithmetic progression is an infinite sequence of numbers where the difference between each two consecutive terms is the same. A geometric progression is an infinite sequence of non-zero numbers where the ratio of each two consecutive terms is the same. We found that for a certain arithmetic progression the 1st, 2nd and 2024th elements give the first three numbers of a certain geometric progression (in that order). Show that all the elements of this geometric progression must appear somewhere in our arithmetic progression.
3. The $mn$ kids in the nursing room sit on $mn$ soft square mats arranged in an $m \times n$ rectangle. Each kid is sitting on a different square, and each kid looks towards one of the sides of the rectangle (they might face in different directions). When the nanny claps her hands, each child crawls over to the next mat in the direction they were looking, and turns 90 degrees to the left or right. If a baby crawls off the mat onto the cold floor, it cries. If two babies end up on the same mat, they cry. For which values of $m$ and $n$ can these crawls last indefinitely without tears?
4. Show that a square with side length 1 cannot be fully covered with three identical disks of diameter 1, but it can be covered with three identical disks of diameter $\frac{101}{100}$.
5. Show that for any positive integer $n \geq 2$ we have
$$2^{2^n(n-2)+n+2} < (2^n)! < 2^{2^n(n-1)+1}.$$
(For a positive integer $M$, $M!$ stands for the product $1 \cdot 2 \cdot 3 \cdots (M-1) \cdot M$.)

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You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

Find old and current problems and information about the talent search at: `http://www.math.wisc.edu/talent`
Find an introduction to techniques for solving problems like these at `https://go.wisc.edu/551pe6`

| **Return To**<br><br>**Or Email To** | **MATHEMATICS TALENT SEARCH**<br>Dept. of Mathematics, 480 Lincoln Drive<br>University of Wisconsin, Madison, WI 53706<br>talent@math.wisc.edu | **Deadline**<br>January 11, 2024 |
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| **Please Fill In** | **PROBLEM SET III** |
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| **Problem** | **Score** |
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