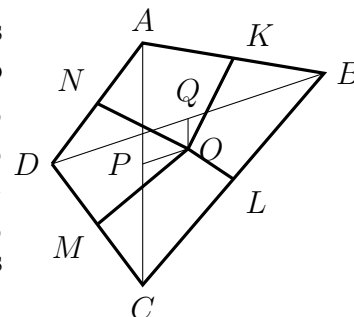


WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET II (2023-2024)

November 2023

- The positive integers 1, 2, 3, through 200 are written down in a list. Alex draws circles around 10 of these numbers, and the mean of Alex's 10 numbers is 40. Beth draws circles around 20 of the numbers, and the mean of Beth's 20 numbers is 70. Show that the mean of the numbers with either one or two circles around them can be 80 but not 82.
- There are 9 pieces of cheese on a plate (possibly with different weights). Is it always possible to cut exactly one of the pieces into two parts so that the resulting 10 pieces can be divided into two groups of 5 pieces each with the two groups having equal weight?
- $ABCD$ is a convex quadrilateral. Denote by P and Q the midpoints of its diagonals. Draw through P and through Q lines parallel to the other diagonal, and let O be their intersection point. Let K , L , M , and N be the midpoints of sides \overline{AB} , \overline{BC} , \overline{CD} , and \overline{DA} , respectively, and draw segments \overline{KO} , \overline{LO} , \overline{MO} , and \overline{NO} , as shown. This subdivides $ABCD$ into four smaller quadrilaterals: $ONAK$, $OKBL$, $OLCM$, and $OMDN$. Prove that these four quadrilaterals all have the same area.
- A regular polygon with 5055 sides has 2023 of its vertices painted blue. Show that there is an isosceles triangle whose vertices are three of these blue vertices.
- Starting with a positive integer, if it is even, divide it by two, but if it is odd, add 7 to it. Call a positive integer *awesome* if applying this step a finite number of times can produce the number 1. For example, 9 is awesome because the steps produce 9, 16, 8, 4, 2, 1. Find the number of awesome positive integers between 1 and 2023.



You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

Find old and current problems and information about the talent search at: <http://www.math.wisc.edu/talent>

Find an introduction to techniques for solving problems like these at <https://go.wisc.edu/551pe6>

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