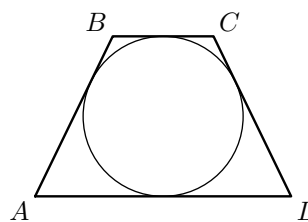


WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET I (2023-2024)

October 2023

- Each of the 2023 guests sitting around a large round table either always tells the truth or always lies. Each guest says “there is exactly one truth-teller sitting next to me.” Show that all the guests are liars.
- Pat and Harper each bicycle uphill from Aston to Benfield and then downhill from Benfield to Coville. Each rider has a constant uphill speed and a constant downhill speed, but Pat climbs hills faster than Harper, and Harper rides downhill faster than Pat. Harper begins the ride 22 minutes before Pat begins, but Pat passes Harper when they are half way between Aston and Benfield. Then Harper passes Pat when they are half way between Benfield and Coville. Which bicyclist completes the ride from Aston to Coville in the least amount of time?
- I wrote down the positive integer number a twice next to each other to get a new number b . (E.g. if $a = 123$, then b would be 123123.) What are the possible values of the fraction $\frac{b}{a^2}$ if we know that the fraction is an integer?



- The four sides of an isosceles trapezoid of area 5 are each tangent to a circle of radius 1. Find the area of the quadrilateral whose vertices are the points of tangency.
- We say that a positive integer *contains 23* if among its digits, we can find 2 and 3 next to each other in that order. Similarly, we say that a positive integer *contains 22* if there are two 2s next to each other among its digits. (E.g. the number 12342 contains 23 but not 22.) Show that there are more 2023-digit integers that contain 23 than those that contain 22.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

Find old and current problems and information about the talent search at: <http://www.math.wisc.edu/talent>

Find an introduction to techniques for solving problems like these at <https://go.wisc.edu/551pe6>

Return To	MATHEMATICS TALENT SEARCH Dept. of Mathematics, 480 Lincoln Drive University of Wisconsin, Madison, WI 53706	Deadline November 10, 2023	
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