

WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET V (2022-2023)

February 2023

1. An empty meeting room has a large round table with 60 chairs around it. A class of 30 students comes into the room, and the students sit down on the chairs around the table (leaving 30 of the chairs empty). Show that one can always find six chairs next to each other so that there are exactly three of them are taken by the students and three of them are empty.
2. Is it possible to color each integer greater than 1 with either red, green, or blue so that all three colors are used, and for every two integers x and y colored in two different colors, the integer $x \cdot y$ is colored with the third color?
3. Each of the following expressions contain 2023 fractions. Which expression has the greater value?

$$\frac{2}{2 + \frac{2}{2 + \frac{2}{2 + \dots + \frac{2}{2}}}} \quad \text{or} \quad \frac{3}{3 + \frac{3}{3 + \frac{3}{3 + \dots + \frac{3}{3}}}}$$

4. Give an example of a hexagon satisfying the following two properties: (a) all the interior angles of the hexagon are either 90 or 270 degrees, and (b) the hexagon can be cut into two unequal hexagons geometrically similar to the original hexagon.
5. A railroad line has 26 stations: A, B, \dots, Z , in this order. A traveler wants to visit each station exactly once, starting and ending at A . He has a lot of time, so he goes from A to Z , then back to B , then to Y , then to C , and so on, until he finally goes from M to N and returns to A . Show that the length of this route is the maximum possible! Note however, that this is not the only route of maximum length; for instance, the traveler could have traced the same itinerary in reverse. How many routes of maximum length are there?

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

Find old and current problems and information about the talent search at: <http://www.math.wisc.edu/talent>

Find an introduction to techniques for solving problems like these at <https://go.wisc.edu/551pe6>

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