WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET IV (2022-2023) **January 2023**

1. Find all positive three-digit integers n such that if s is the sum of its digits, all digits of the number $n+s$ are equal, and all digits of $n-s$ are also equal (but possibly distinct from the digits of $n+s$).

2. A collection of 2023 real numbers has the property that its mean, median, and range all equal 2023. What is the greatest possible value of its maximum?

(If the numbers are, in order, $a_1 \geq a_2 \geq \cdots \geq a_{2023}$, the *mean* is $\frac{1}{2023}(a_1 + \cdots + a_{2023})$, the *median* is a_{1012}, and the *range* is $a_1 - a_{2023}$.)

3. Let W_1 and W_2 be disjoint circles with radii r_1 and r_2, respectively. (*Disjoint* means that W_1 and W_2 are not allowed to intersect, or to be tangent.) Show that there are two distinct concentric circles that are each tangent to both W_1 and W_2 if and only if $r_1 = r_2$.

4. We have a collection of 2023 squares of various sizes and of total area 4. Show that we can use this collection to fully cover a unit square (overlaps are allowed).

5. (This problem concerns the same game as Problem 5 from the previous problem set.) Two players, Angelica and Brian, each have a rectangular 20×21 board and 140 identical 1×3 tiles ('straight trominos'), enough to completely cover their board with no overlaps. First, Angelica covers her board with her tiles (with no overlaps), and then Brian looks at what she did and covers his board with his tiles. Angelica gets a point for each tile that is in exactly the same position in the two tilings. Show that Brian has a strategy such that Angelica gets at most 14 points for every one of Angelica's tilings.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

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Return To **Or Email To**	**MATHEMATICS TALENT SEARCH** Dept. of Mathematics, 480 Lincoln Drive University of Wisconsin, Madison, WI 53706 talent@math.wisc.edu	**Deadline** February 13, 2023

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