

WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET III (2022-2023)

December 2022

1. The Bureau of Weights and Measures had a collection of weights of 1 gram, 2 grams, . . . , 2023 grams. When the Bureau moved to a new building, the 100 gram weight was lost. Is it possible to distribute the remaining weights between two storage cabinets so that the total weight in each cabinet is the same?
2. Find the greatest positive integer  $n$  so that the number  $n! \cdot 2022!$  can be expressed as  $k!$  for some positive integer  $k$ .
3. The difference between the roots of a quadratic polynomial  $P(x)$  is equal to 2, and the difference between the roots of  $P(x) + 3$  is equal to 4. Find the difference between the roots of  $P(x) + 8$ .
4.  $\triangle ABC$  is a right triangle with a right angle at  $B$ . The point  $P$  is on the side  $\overline{AB}$  and the point  $Q$  on the side  $\overline{BC}$  so that  $AP = CB$  and  $BP = CQ$ . Show that the angle between  $\overline{AQ}$  and  $\overline{CP}$  is equal to  $45^\circ$ .
5. Two players, Angelica and Brian, each have a rectangular  $20 \times 21$  board and 140 identical  $1 \times 3$  tiles ('straight trominos'), enough to completely cover their board with no overlaps. First, Angelica covers her board with her tiles (with no overlaps), and then Brian looks at what she did and covers his board with his tiles. Angelica gets a point for each tile that is in exactly the same position in the two tilings. Show that Angelica has a strategy for getting at least 14 points regardless of Brian's tiling.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

Find old and current problems and information about the talent search at: <http://www.math.wisc.edu/talent>

Find an introduction to techniques for solving problems like these at <https://go.wisc.edu/551pe6>

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