

# WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

## PROBLEM SET II (2022-2023)

November 2022

1. John picked an arbitrary positive integer, multiplied it by 5, multiplied the result by 5, then multiplied the result by 5 again and so on. Is it true that from some point on, all the numbers that John obtains contain a 5 in their decimal representation?
2. Call a collection of different positive integers between 1 and 999 *good* if 1000 can be represented as a sum of several different integers from the collection. Call such a collection *bad* if it is not good. For instance,  $\{1, 2, 299, 300, 400\}$  is a good collection, because  $1 + 299 + 300 + 400 = 1000$ , but  $\{5, 6, 7, 8, 9\}$  is a bad collection. What is the maximum possible number of integers in a bad collection?
3. Find the least positive integer  $n$  so that the product of all of the positive divisors of  $n$  (including  $n$  itself) is equal to  $n^{199}$ .
4. Let  $m, n, r,$  and  $s$  be positive integers satisfying  $m + n = r + s$  and  $r < m \leq n < s$ . Show that

$$(2^r + 3^r)(2^s + 3^s) > (2^m + 3^m)(2^n + 3^n).$$

5. The five sides of a convex pentagon  $ABCDE$  all have the same length, and the three diagonals  $\overline{AD}$ ,  $\overline{AC}$ , and  $\overline{BE}$  all have the same length. Is it necessary that the pentagon is regular?

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

Find old and current problems and information about the talent search at: <http://www.math.wisc.edu/talent>

Find an introduction to techniques for solving problems like these at <https://go.wisc.edu/551pe6>

<b>Return To</b>	<b>MATHEMATICS TALENT SEARCH</b> Dept. of Mathematics, 480 Lincoln Drive University of Wisconsin, Madison, WI 53706	<b>Deadline</b> December 12, 2022	
<b>Or Email To</b>	talent@math.wisc.edu		
<b>Please Fill In</b>	<b>PROBLEM SET II</b>	<b>Problem</b>	<b>Score</b>
Name & Grade		1	
School & Town		2	
Home Address		3	
Town & Zip		4	
Email Address		5	
Teacher's Name			
Teacher's Email			