

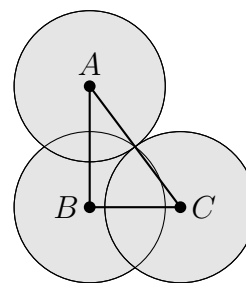
WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH
PROBLEM SET I (2022-2023) **October 2022**

- The last Badger game had 10% more people in attendance than the previous game, but the number of empty seats was 15% less than during the previous game. What percentage of seats was occupied during the last Badger game, assuming all the people in attendance took their seats during the game.
- Let $n > 2$ be an integer, and consider the fractions $\frac{1}{n}, \frac{2}{n}, \dots, \frac{n-1}{n}$. Show that the number of irreducible fractions in this collection is even. (A fraction is called irreducible if the numerator and the denominator do not have common integer divisors other than ± 1 .)
- Show that if x, y , and z are nonzero numbers satisfying the equations

$$\frac{x+y}{x^2+y^2} = \frac{y+z}{y^2+z^2} = \frac{z+x}{z^2+x^2},$$

then we must have $x = y = z$.

- Triangle $\triangle ABC$ has sides of lengths $AB = 16$, $BC = 12$, and $CA = 20$. Three circles, all of radius 10, are centered at points A , B , and C . Find the total area covered by the three circles (the shaded area in the picture).



- Someone chose 8 different positive integers no greater than 16. Show that among all of their positive pairwise differences there are at least 3 that are equal.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

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