

WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET V (2021-2022)

February 2022

1. In how many ways can you mark 8 of the 16 cells in a  $4 \times 4$  grid of cells so that there are 2 marked cells in each row, 2 marked cells in each column, and neither of the diagonals have all 4 of their cells marked?
2. A triangle  $ABC$  has a point  $K$  on the side  $AB$  and a point  $L$  on the segment  $CK$  such that  $AK = KL = \frac{1}{2}KB$ . Show that if  $\angle CAB = 45^\circ$  and  $\angle CKB = 60^\circ$ , then  $AL = BL = CL$ .
3. We tiled the plane perfectly with  $1 \times 1$  unit squares, forming an infinite grid. Taylor removed finitely many of these tiles, with no two of the removed tiles having common vertices or edges. Is it true that the remaining tiles can always be covered with non-overlapping  $2 \times 1$  dominoes? (Each domino has to cover exactly two unit squares.)
4. The function  $f$  is defined for all real  $x$  other than  $x = 0$  and  $x = 1$  and satisfies the equation

$$f(x) + f\left(1 - \frac{1}{x}\right) = 2^x$$

for all  $x$  where it is defined. Find a formula for  $f(x)$ .

5. Show that if  $x$  and  $y$  are real numbers, then we can always find integers  $m$  and  $n$  with

$$(x - m)^2 + (y - n)^2 + (x - m)(y - n) \leq \frac{1}{3}.$$

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

Find old and current problems and information about the talent search at: <http://www.math.wisc.edu/talent>

Find an introduction to techniques for solving problems like these at: <https://go.wisc.edu/551pe6>

<b>Return To</b>	<b>MATHEMATICS TALENT SEARCH</b> Dept. of Mathematics, 480 Lincoln Drive University of Wisconsin, Madison, WI 53706 talent@math.wisc.edu	<b>Deadline</b> March 6, 2022	
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