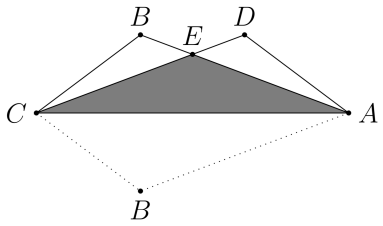
WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET III (2021-2022) **December 2021**

1. A parallelogram $ABCD$ has sides $AB = 7$ and $BC = 3$. When the parallelogram is folded along its long diagonal $\overline{AC}$, the sides $\overline{AB}$ and $\overline{CD}$ cross at point E, and the area of $\triangle ACE$ is the same as the sum of the areas of $\triangle ADE$ and $\triangle CBE$. Find the length of the diagonal $\overline{AC}$.



2. Find the greatest integer n for which $n + 2021$ divides n^{2022}.

3. For which positive integer k will the expression $\frac{1776^k + 2022^k}{k!}$ take its greatest possible value? ($k! = k \cdot (k-1) \cdots 2 \cdot 1$ is k factorial.)

4. Keisha drew 100 lines on the plane, with no three lines passing through the same point. These lines divide up the plane into various parts, and some of these parts are triangle-shaped. Jay claims that he can draw another line that does not pass through any of the existing intersection points, and it intersects at least 60 triangle-shaped parts. Show that Jay's claim cannot be true.

5. Samir has 1000 cards, each labeled with a different three digit sequence from $(0, 0, 0)$ to $(9, 9, 9)$. He also has 100 boxes that are labeled with different two digit sequences from $(0, 0)$ to $(9, 9)$. He wants to place each of the cards into a box so that the label of the box is the same as the label of the card with one digit deleted. (So the card $(1, 2, 3)$ can be placed in any of the boxes $(1, 2)$, $(2, 3)$ and $(1, 3)$, but the card $(0, 0, 0)$ can only be placed in box $(0, 0)$.) Show that Samir can place the cards into 50 of the boxes, but he cannot place them into fewer than 50.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

Find old and current problems and information about the talent search at: `http://www.math.wisc.edu/talent`

Find an introduction to techniques for solving problems like these at: `https://go.wisc.edu/551pe6`

Return To **Or Email To**	**MATHEMATICS TALENT SEARCH** Dept. of Mathematics, 480 Lincoln Drive University of Wisconsin, Madison, WI 53706 talent@math.wisc.edu

Deadline
January 4, 2022

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