

WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET II (2021-2022)

November 2021

- Two circles of radius 1 are tangent at the point P . Let A be a point on one of the circles and let B be a point on the other circle such that the angle $\angle APB$ is 90 degrees. Find the length of AB .
- For a real number x , we denote by $\lfloor x \rfloor$ the greatest integer less than or equal to x . (This is called the floor function.) For example, $\lfloor 2.3 \rfloor = 2$, $\lfloor -4.2 \rfloor = -5$, and $\lfloor 7 \rfloor = 7$. Find all integer solutions of the following equation:

$$\lfloor x/3 \rfloor = \lfloor x/4 \rfloor + 1.$$

- Let $p(n) = n^3 - 12n^2 + 32n + 3$. Note that $p(4)$ is a prime number. Find all the integers n such that $p(n)$ is a (positive) prime number.
- We write the numbers $1, 2, 2^2, 2^3, \dots, 2^{2021}$ on a (very large) blackboard. We are allowed to make the following changes to these numbers: we can erase (any) two numbers on the board, and then write the absolute value of their difference on the board. We repeat this process 2021 times, after which we have only one number remaining on the board. Show that the last number has to be an odd integer between 1 and $2^{2021} - 1$, and, conversely, that any odd integer between 1 and $2^{2021} - 1$ can be obtained by this procedure.
- There are 50 girls and 50 boys standing in line in some order. We know that there is exactly one stretch of 30 children next to each other with an equal number of boys and girls. Show that there is also a stretch of 70 children in a row with an equal number of girls and boys.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

Find old and current problems and information about the talent search at: <http://www.math.wisc.edu/talent>

Find an introduction to techniques for solving problems like these at: <https://go.wisc.edu/551pe6>

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