WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET I (2021-2022)   **October 2021**

1. We have four identically looking gold chains with weights 1, a, a^2, and a^3, where $a > 1$ is an unknown value. We also have a balance scale with two pans that can be used to compare weights. When used, the scale only shows which side is heavier, or that the two sides weight the same; it does not show the actual weights. Find a way to identify the heaviest of the four chains by using two comparisons. (You can decide which and how many chains you put in the pans.)

2. Each square of a 100×100 board is painted in one of 20 available colors. Call a square *lonely* if its color is different from all other squares in its row and in its column. What is the maximal possible number of lonely squares on the board?

3. In an acute isosceles triangle one of the angles is n times as large as another one, where $n > 1$ is an integer. Show that the triangle can be cut into n smaller triangles with $n - 1$ straight cuts so that all the resulting smaller triangles are isosceles.

4. Find all integers d for which there are infinitely many integer solutions (x, y) of the equation
$$x^2 + x + 2021 = y^2 + 3y + d.$$

5. Let P be a polygon and L be a line such that L intersects P in exactly 2021 points. Show that there is another line that intersects P in more than 2021 points. (A polygon is a finite number of straight line segments that connect end-to-end to form a closed path that does not intersect itself.)

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

Find old and current problems and information about the talent search at: `http://www.math.wisc.edu/talent`

Find an introduction to techniques for solving problems like these at `https://go.wisc.edu/551pe6`