

WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET V (2020-2021)

February 2021

- Jodie chooses at most 7 prime numbers which are greater than 10, computes the sum of their squares, and tells this sum to Emily. Show that Emily can always identify the number of primes Jodie chose. The primes that Jodie chooses do not have to be different, and Emily does not need to identify the actual primes chosen by Jodie.
- Maya and Yolanda take turns writing positive integers that are at most 1000 on a large whiteboard, with Maya going first. A player is not allowed to write the number 13, to write a number which has already appeared on the board, or to write a number which differs by 17 from a number which has already appeared on the board. The first player who cannot write another number loses. Which player has a winning strategy? (You also have to provide a possible winning strategy.)
- Find the smallest integer $n \geq 2$ such that any rectangle can be divided into n non-overlapping smaller rectangles (with sides parallel to the original one) in a way that no pair of smaller rectangles has two vertices in common. (You must provide a construction for n , and prove that $n - 1$ is impossible.)
- A point P is chosen on the side BC of a quadrilateral $ABCD$. The circumcircles of the triangles $\triangle PAB$ and $\triangle PCD$ are both tangent to AD , and $\angle BAP = \angle PDC = 30^\circ$. Find $\angle APD$.
- Find the integer K such that

$$K \leq \frac{1}{\sqrt{2^{2020}}} + \frac{1}{\sqrt{2^{2020} + 1}} + \frac{1}{\sqrt{2^{2020} + 2}} + \cdots + \frac{1}{\sqrt{2^{2024} - 1}} < K + 1.$$

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

Find old and current problems and information about the talent search at: <http://www.math.wisc.edu/talent>

Find an introduction to techniques for solving problems like these at: <https://goo.gl/pqq32m>

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Deadline March 6, 2021	
Problem	Score
1	
2	
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