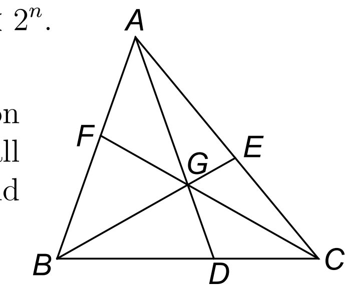
WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET III (2015-2016) **December 2015**

1. We have a list of n different numbers with $n > 1$. We write down the sum of every pair of distinct numbers in our list. Show that we must have at least $2n - 3$ different sums.
2. We have n numbers $x_1, \ldots, x_n$ such that $0 < x_i < 1$ for each i, and $n > 1$. Show that
$$2 < (1 - x_1)(1 - x_2) \cdots (1 - x_n) + (1 + x_1)(1 + x_2) \cdots (1 + x_n) < 2^n.$$
3. In $\triangle ABC$, $AB = 9$, $BC = 10$, and $AC = 11$. Points D, E, and F lie on sides $\overline{BC}$, $\overline{AC}$, and $\overline{AB}$, respectively, so that segments $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ all intersect at G as shown. Given that $BF = CE$ and $AG : DG = 2 : 1$, find BF.



4. We define a sequence of integers $a_1, a_2, \ldots, a_n, \ldots$ successively, by setting $a_1 = 2$ and $a_{n+1} = a_n^2 - a_n + 1$ for $n \geq 1$. The first few elements are $a_1 = 2, a_2 = 3, a_3 = 7, a_4 = 43$. Show that in the sequence we cannot find two different integers with a common divisor that is bigger than one.
5. Anne and Bert play the following game. They start with a $1 \times n$ grid of n unit squares which are initially white. Anne starts the game, and they take steps one after another. In each step the appropriate player can change either one or two neighboring white squares to black. The player who colors the last available square(s) wins. For which n will Anne have a winning strategy?

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

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