

WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH
PROBLEM SET IV (2020-2021) **January 2021**

1. We draw the angle bisectors of two of the angles in a triangle. (The angle bisectors are rays that divide the corresponding angles into two equal parts.) These two bisectors intersect at a 60° angle. Show that the triangle must have a 60° angle.
2. Sandy buys plain white plates and paints designs on them. For an 80-day period last summer, she painted either 4, 5, 6, or 7 plates every day. Whenever Sandy completed a set of four plates, she put them in a box to sell. Show that there was a sequence of consecutive days last summer when Sandy boxed exactly 19 sets of plates.
3. Farmer John has 26 pounds of feed to distribute among 35 cows. Prove that regardless of the initial weights of the cows, he can always distribute the feed in such a way that the sum of the weights of every two cows after feeding will be an integer. Assume that every cow eats its entire allotment of food, and the weight of a cow after feeding is the sum of its initial weight and the weight of food it consumes. Farmer John need not use all 26 pounds of food.
4. Let $x_1, x_2, \dots, x_{2021}$ be 2021 distinct numbers. A number S is called *special* if it can be written as

$$S = 1 \cdot y_1 + 2 \cdot y_2 + \dots + 2021 \cdot y_{2021},$$

where the numbers y_1, \dots, y_{2021} are the numbers $x_1, x_2, \dots, x_{2021}$ rearranged in some order. Show that there are at least $2021 \cdot 1010$ different special numbers.

5. Suppose that the four faces of a tetrahedron all have the same area. Show that the four faces must be congruent to each other.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

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