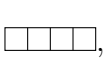
WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET III (2020-2021) **December 2020**

1. We have a 100×100 board built from 10,000 unit squares. We have an infinite supply of the following types of tiles, each composed of four unit squares:  ,  ,  ,  ,  . We fully covered our board with these tiles (possibly rotating or flipping some of them, but without overlaps), and then counted the number of tiles used to cover each row and column. The total of these 200 numbers was 12,000. How many 2×2 square tiles did we use?

2. Suppose $f(x)$ and $g(x)$ are quadratic polynomials, i.e., $f(x) = ax^2 + bx + c$ and $g(x) = px^2 + qx + r$, where a, b, c, p, q, and r are numbers. We know that the equation $f(x) = g(|x|)$ has four distinct real-valued solutions. How many distinct real-valued solutions does the equation $f(|x|) = g(x)$ have?

3. Call a group of 31 integers *nice* if any 12 of the 31 integers can be divided into two groups of 6 so that the sum is the same in the two groups. Show that if 31 integers form a nice group, then they are all equal to each other.

4. Suppose for some integer $k > 1$ that $d_1 < \cdots < d_k$ are positive integers, and M is a positive integer that is divisible by all of them. Prove that

$$d_1d_2 + d_2d_3 + d_3d_4 + \cdots + d_{k-1}d_k + d_kd_1 \leq M^2.$$

5. Show that if we have 13 points in the plane, we can always choose three of the 13 points that are not the vertices of an isosceles triangle.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

Find old and current problems and information about the talent search at: `http://www.math.wisc.edu/talent`

Find an introduction to techniques for solving problems like these at: `https://goo.gl/pqq32m`

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