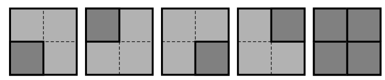
WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET II (2020-2021) **November 2020**

1. Find all positive integer pairs (m, n) satisfying $m^2 - n^2 - 2n = 2021$.

2. Show that for every nonnegative number x, we have $x + 3 \geq 4\sqrt[4]{x}$.

3. We have a $2 \times n$ board made up of $2n$ unit squares. We also have an unlimited supply of two types of tiles: 1×1 tiles, and L-shaped tiles composed of three 1×1 tiles. Let a_n denote the total number of ways to fully cover the $2 \times n$ board with the available tiles in a single layer. (E.g., $a_2 = 5$, as demonstrated in the figure below.) Prove that $a_n \leq 3^n$.



4. A strange new planet has been been discovered, shaped like a regular tetrahedron with edge length 900 miles. (A regular tetrahedron has four faces which are all equilateral triangles.) The entire surface of the planet is covered by oceans. Astronomers observe that a massive earthquake has triggered a tsunami whose waves are traveling along the surface at the speed of 300 miles per hour. If the epicenter of the earthquake is at the centroid of a face, how long does it take for the waves to reach the vertex opposite to this face?

5. We have 2020 distinct numbers arranged in a circle. We say that a pair of numbers A, B is *dominating* if A and B are not next to each other on the circle, and on one of the two arcs between A and B, all numbers are smaller than both A and B. (If all numbers on both arcs happen to be smaller than A and B, we still call the pair dominating.) Find the number of dominating pairs, and show that the answer is the same, no matter how the numbers are arranged.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

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