WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET I (2020-2021) **October 2020**

1. You have 2020 \$1 bills and 11 envelopes. Show that it is possible to divide the bills among the envelopes in such a way that you could give someone any exact integer amount from \$1 to \$2020 by handing over some of the envelopes (at least one, at most 11). Then show that this would be impossible if you only had 10 envelopes.

2. In a group of 300 people, each person is friends with exactly 200 others. (Nobody is their own friend, and if A is B's friend, then B is A's friend.) However, no four people are all friends with each other. Show that it is possible to find a group of 100 people in which no two are friends.

3. Prove that any non-equilateral triangle can be divided into four similar triangles such that the four triangles are not all congruent to each other.

4. Three positive numbers a, b, and c have a sum of 1. Show that

$$(ab + bc + ac)^2 \geq 3abc.$$

5. Find all positive integers a and b such that both $a^2 - 4b$ and $b^2 - 4a$ are perfect squares.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

Find old and current problems and information about the talent search at: http://www.math.wisc.edu/talent
Find an introduction to techniques for solving problems like these at: https://goo.gl/pqq32m

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