WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET V (2019-2020) **February 2020**

1. The product of four consecutive integers $a, a+1, a+2$ and $a+3$ can also be written as a product of two consecutive integers. What are the possible values of a?
2. At time 0, a Petri dish contains one red and one blue amoeba. At the end of each minute, exactly one amoeba in the dish divides into two identical amoebas, each with the same color as its parent amoeba. Thus, at the end of n minutes, the dish contains $n+2$ amoebas. Assume that the amoeba that divides into two is always chosen uniformly at random (i.e., it is equally likely that any amoeba is chosen). Find the probability that after 2020 minutes, the Petri dish contains more red than blue amoebas.
3. Suppose that a point P inside a given convex quadrilateral $ABCD$ has the property that the areas of the triangles PAB, PBC, PCD, and PAD are all equal. Show that one of the diagonals of $ABCD$ must divide the quadrilateral into two triangles with equal areas.
4. A class of n students has seven clubs. A student can be a member of multiple clubs, and each club has at least one member. Somebody noticed that if we choose any 4 of the 7 clubs, each of the n students is a member of at least one of the 4 clubs. However, for any of choice of 3 of the 7 clubs, there is a student who is not a member of any of the 3 clubs. What is the smallest possible value of n?
5. A game is played on a 100×100 board. Ramon controls two white game pieces which start in the bottom left and top right corners. Josie controls two black game pieces which start in the bottom right and top left corners. The players move alternately. In each move, a player can move one of the game pieces under control to a vacant square which shares a common side with its current location. Ramon wins if he can move the two white game pieces to be next to each other (i.e., located in two squares with a common side) within his first 1000 moves; otherwise, Josie wins. Who has a winning strategy?

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

Find old and current problems and information about the talent search at: `http://www.math.wisc.edu/talent`

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