WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET IV (2019-2020)   January 2020

1. We are given a 2020-sided convex polygon. We want to select three distinct edges of the polygon, so that if we go around the edges in clockwise order, at least two unselected edges lie between every pair of selected edges. In how many different ways can we select the three edges?

2. On quadrilateral $ABCD$, points E, F, G, and H are the midpoints of sides $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$, respectively. Suppose the diagonals $\overline{AC}$ and $\overline{BD}$ of quadrilateral $ABCD$ intersect at the same point as the diagonals $\overline{EG}$ and $\overline{FH}$ of quadrilateral $EFGH$. Show that $ABCD$ must be a parallelogram.

3. In triangle $\triangle PQR$, the midpoint of side $\overline{QR}$ is denoted by S. Find $\angle QPR$, if we know that $\angle PRQ = 30°$ and $\angle PSQ = 45°$.

4. We construct a sequence of prime numbers $p_1, p_2, \ldots$, as follows: We set $p_1 = 2$. For any $n \geq 1$, the integer p_{n+1} is the largest prime factor of the number which is one larger than the product of $p_1, \ldots, p_n$. (So for example, p_4 is the largest prime factor of $1 + p_1 \cdot p_2 \cdot p_3$.) Show that $p_n \neq 5$ for all $n \geq 1$.

5. The edge lengths of a triangle are given by a, b, and c. We know that $ab + bc + ac = 12$. Show that the perimeter of the triangle cannot be larger than 7.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

Find old and current problems and information about the talent search at: `http://www.math.wisc.edu/talent`
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