

**WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH
 PROBLEM SET III (2019-2020)**

December 2019

1. Brianna has a map of the 48 contiguous states of the United States. (These are the states without Alaska and Hawaii.) She wants to color these states with three colors (green, yellow, or brown) so that each state is colored with exactly one of these colors, and no state with the color green shares a border with a state that is colored yellow. Let N denote the number of such possible colorings. Prove that N is an odd number.
2. The numbers $1, 2, \dots, 2019$ are arranged in an arbitrary order $x_1, x_2, \dots, x_{2019}$. (Here, x_i denotes the i th number.) Show that there exist distinct indices i and j with $1 \leq i < j \leq 2019$ such that $|x_i - i| = |x_j - j|$.
3. A pentagon $A_1A_2A_3A_4A_5$ is inscribed in a circle of radius 10. The points B_1, B_2, \dots, B_5 are chosen on the sides of the pentagon $\overline{A_1A_2}$, $\overline{A_2A_3}$, $\overline{A_3A_4}$, $\overline{A_4A_5}$, and $\overline{A_5A_1}$, respectively. Given that the *area* of the pentagon $A_1A_2A_3A_4A_5$ equals 200. Prove that the perimeter of the pentagon $B_1B_2 \dots B_5$ is at least 40.
4. For each number n , denote by a_n the integer closest to \sqrt{n} . This means that a_n is obtained by rounding \sqrt{n} up or down in the usual way; for instance, $\sqrt{2} \approx 1.41$, so $a_2 = 1$, and $\sqrt{3} \approx 1.73$, so $a_3 = 2$. Find the sum $\frac{1}{a_1} + \frac{1}{a_2} + \dots + \frac{1}{a_{2019}}$.
5. We want to reach the number 2020 from the number 1 by performing a sequence of operations. In each step, we can either triple our number or add 1 (e.g., we could triple 1 six times to obtain $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 729$, and then add one 1291 times to arrive at 2020). Find the minimal number of operations needed to obtain 2020 from 1. Make sure to prove that your strategy reaches 2020 in the smallest number of steps.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

Find old and current problems and information about the talent search at: <http://www.math.wisc.edu/talent>

Find an introduction to techniques for solving problems like these at: <https://goo.gl/pqq32m>

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