

WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET II (2019-2020)

November 2019

1. Is there an integer n so that the sum of its digits is equal to the sum of the digits of n^2 , and the latter sum exceeds 2019?
2. Show that for any positive integer k , there is a perfect square among the $k + 1$ consecutive integers $k, k + 1, \dots, 2k$.
3. A regular decagon (ten-sided polygon) has five arbitrarily chosen vertices colored red. Show there exists an isosceles triangle whose vertices are three of these five red vertices.
4. 2019 rocks, all of different weights, are placed in a circle. Between each adjacent pair of rocks lies a ball with weight equal to the difference between the weights of the two rocks. Show that the 2019 balls can be distributed among the two pans of a balance scale so that the pans perfectly balance.
5. We know that a convex polygon can be divided into $30^\circ - 60^\circ - 90^\circ$ triangles which are not necessarily congruent to each other. What is the maximum number of sides such a polygon can have? (You must provide a construction to prove that such a polygon with this number of sides exists, and also show that this is the maximal such number.)

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

Find old and current problems and information about the talent search at: <http://www.math.wisc.edu/talent>

Find an introduction to techniques for solving problems like these at: <https://goo.gl/pqq32m>

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