

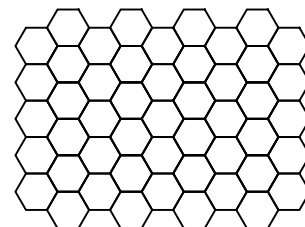
WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET IV (2018-2019)

January 2019

1. How many 5-digit integers  $\underline{ABCDE}$  can we find so that  $\underline{AB}$ ,  $\underline{BC}$ ,  $\underline{CD}$ , and  $\underline{DE}$  are all 2-digit squares? Here,  $\underline{A}$ ,  $\underline{B}$ ,  $\underline{C}$ ,  $\underline{D}$ , and  $\underline{E}$  denote digits that are not necessarily distinct.
2. In the triangle  $\triangle ABC$ , we have  $AB = 20$ ,  $AC = 21$ , and  $BC = 29$ . The points  $E$  and  $D$  are on the side  $\overline{BC}$  so that  $BD = 8$  and  $EC = 9$ . Compute the angle  $\angle DAE$ .

3. A plane is tiled with regular hexagons. This tiling is sometimes called the “honeycomb” lattice. (The picture on the right shows a finite portion of the tiling.) Show that if a line passes through two points that are vertices of hexagons in the tiling, then the line passes through infinitely many such points.



4. We know that  $a$  and  $b$  are positive numbers that satisfy  $1/4 < a(1-b)$ . From this information, is it possible to determine which of the numbers  $a$  or  $b$  is larger?
5. We call a positive integer “interesting” if each one of its digits is either 0, 1, or 2, and any two of its neighboring digits differ by at most one. (For example, the number 1210012 is interesting, but the number 1200 is not.) Show that the number of  $n$ -digit interesting numbers is at most  $(\frac{5}{2})^n$ .

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

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