

WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET III (2018-2018)

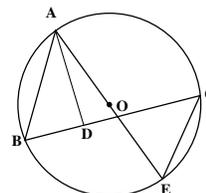
December 2018

1. Is it possible to find distinct positive integers  $B, U, C, K,$  and  $Y,$  which satisfy the equality on the right?

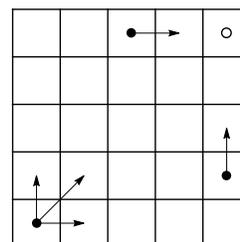
$$B + \frac{1}{U + \frac{1}{C + \frac{1}{K + \frac{1}{Y}}}} = \frac{22703}{7177}$$

2. Show that the numerical value of the expression  $\frac{2 \cdot 4 \cdot 6 \cdots 2018 \cdot 2020}{1 \cdot 3 \cdot 5 \cdots 2017 \cdot 2019}$  is between 44 and 64.

3. The points  $A, B, C,$  and  $E$  lie on the boundary of a circle with center  $O.$  The point  $D$  lies on the line segment  $BC,$  and  $AD$  is perpendicular to  $BC.$  We know that  $AB = 12, BD = 6$  and  $CE = 8.$  Find the radius of the circle.



4. Ariana and Brooke play a game. They have a game board with an  $n \times n$  grid of squares and a game piece in the bottom left square. The two players move the game piece one after another, beginning with Ariana. In each move, the game piece can be moved one step either up, right, or in the up-right diagonal direction, as long as it stays on the board. The first player to move the game piece into the top right corner square wins the game. For which  $n > 1$  will Ariana have a winning strategy?



5. A cube has one red and five white faces. We place it on a table so that it rests on the red face. An “edge move” is a  $90^\circ$  rotation of the cube around one of the four edges resting on the table, so that a new face will then be resting on the table. Suppose somebody performs 12 edge moves, so that each one of the cube’s 12 edges is used exactly once as an axis of rotation. Prove that after the final move, the cube will again rest on the red face.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

Find old and current problems and information about the talent search at: <http://www.math.wisc.edu/talent>

Find an introduction to techniques for solving problems like these at: <https://goo.gl/pqq32m>

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