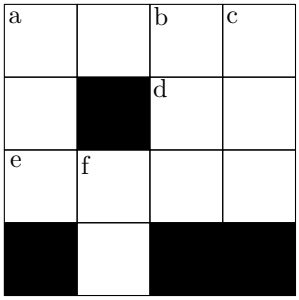
WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET I (2018-2019) **October 2018**

1. Find all solutions to the "crossnumber" puzzle using the given clues.



Across:
(a) Sum of the digits in the number is 11.
(d) Square of a prime number.
(e) Consecutive digits in descending order.

Down:
(a) Fourth power of a number.
(b) Square of a number.
(c) Product of the digits in the number is 216.
(f) Multiple of 11 that ends in 5.

2. Triangle ABC has angles $\angle ABC = 90^o$ and $\angle BCA = 60^o$. Show that $\triangle ABC$ can be cut into three congruent triangles.

3. Suppose that $r > 2$ is an integer. Show that $\sqrt{r} - \sqrt{2}$ cannot be an integer.

4. Let G be a point in the interior of the triangle ABC, and let D, E, and F be points on sides $\overline{BC}$, $\overline{AC}$, $\overline{AB}$, respectively, such that $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ all intersect at G. Given that $\triangle AFG$, $\triangle AGE$, and $\triangle BDG$ all have the same area, show that D, E and F are the midpoints of the sides $\overline{BC}, \overline{AC}$, and $\overline{AB}$, respectively.

5. The numbers 1,2,..., 2018 are written in order in a long line on a big board. Somebody plays the following game: she chooses two numbers next to each other, erases them, and writes the absolute value of the difference of the two numbers in place of them (only once). She repeats this until only one number is left on the board. Find all possible numbers that can be equal to this final number.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

Find old and current problems and information about the talent search at: `http://www.math.wisc.edu/talent`

Find an introduction to techniques for solving problems like these at: `https://goo.gl/pqq32m`

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