

# WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

## PROBLEM SET III (2017-2018)

December 2017

1. Find the greatest integer that is exactly 19 times the sum of its digits.
2. The sum of 100 distinct positive integers is 5052. What is the value of the 5th largest number?
3. How many right triangles are there whose legs have integer lengths and whose hypotenuse has length  $2^{2017} \cdot \sqrt{2}$ ?
4. It's Christmas Eve, and Santa is having a busy night, as usual. After making his way down a particularly narrow chimney, Santa is delighted to find some refreshments for him—a glass of eggnog and a glass of mulled wine. Santa notices that there are exactly 300 mL of eggnog and 200 mL of mulled wine. The eggnog contains 5% alcohol by volume (i.e., every 100 mL of eggnog contains 5 mL of alcohol), and the mulled wine contains 10% alcohol by volume. Furthermore, each mL of eggnog contains 2 calories and each mL of the mulled wine has 3 calories. Santa, being hungry, wants to consume as many calories as possible. However, being a responsible reindeer-sleigh driver, he wants to limit his alcohol intake to at most 25 mL. How much eggnog and mulled wine should he drink (in mL) so that he maximizes his caloric intake?
5. Suppose that  $ABCD$  is a convex quadrilateral. Show that we can find a parallelogram which covers  $ABCD$  and has at least three vertices in common with  $ABCD$ .

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

Find old and current problems and information about the talent search at: <http://www.math.wisc.edu/talent>

Find an introduction to techniques for solving problems like these at: <https://goo.gl/pqq32m>

<b>Return To</b>	<b>MATHEMATICS TALENT SEARCH</b> Dept. of Mathematics, 480 Lincoln Drive University of Wisconsin, Madison, WI 53706	<b>Deadline</b> January 6, 2018	
<b>Or Email To</b>	talent@math.wisc.edu		
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Please check here if you would like to be emailed when new problem sets are available.