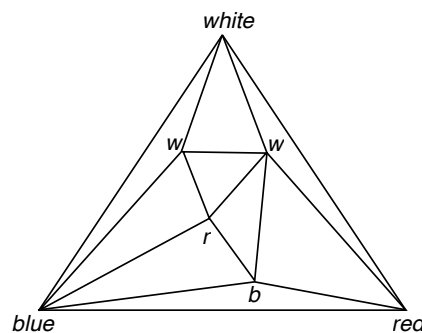


WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH
PROBLEM SET II (2017-2018)

November 2017

1. Consider the number $a = 20172017\dots2017$ where the digits 2,0,1,7 repeat one thousand times. In this number only four of the possible 10 digits appear. Show that the number a has an integer multiple that contains all ten digits in its decimal representation.
2. How many ways can you paint the six faces of a cube using the colors red, blue, yellow, green, pink, where each face is painted with a single color, and all 5 colors must be used at least once? Two colorings are considered the same if one may be obtained from the other by rotating the cube.
3. A quadrilateral $ABCD$ has side lengths $AB = BC = 3$ and $CD = DA = 5$. Let P be a point on the line segment AC and compute the distance of P from each of the four sides. Show that the sum of these four distances does not depend on the position of P .
4. How many positive real numbers x satisfy $\frac{\sqrt{x+2017}}{100} = x - \lfloor x \rfloor$? Here $\lfloor x \rfloor$ refers to the greatest integer less than or equal to x . (E.g. $\lfloor 3.9 \rfloor = 3$ and $\lfloor 5 \rfloor = 5$.)
5. The United Triangular States is a triangle shaped country with a red flag, a white flag, and a blue flag marking its three corners. The country is divided into a number of triangle shaped states. No vertex of any state is on another state's border or on the country's border unless it is also a vertex of that other state or the country. (See the figure for an example.) Every vertex is marked with a red, white, or blue flag. Prove that no matter how the map is drawn and how the flag colors are assigned, there must be at least one state whose three vertices are marked with all three colors.



You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

Find old and current problems and information about the talent search at: <http://www.math.wisc.edu/talent>

Find an introduction to techniques for solving problems like these at: <https://goo.gl/pqq32m>

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