

WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET I (2017-2018)

October 2017

1. During a certain calendar year there were three months in a row so that each month had exactly 4 Saturdays. Find all possible choices for three consecutive months in which this can happen. (Make sure you justify why you have found all the possibilities.)
2. Find 2017 positive integers (not necessarily distinct), so that their sum is the same as their product.
3. On each side of a triangle construct a circle where the center of the circle is the midpoint of the side, and the radius of the circle is $\frac{1}{4}$ the length of the side. Show that if for each pair of two circles there is at least one point common to both circles, then the triangle must be an equilateral triangle.
4. 10 players play a round robin tournament. This means that each player plays with every other player exactly once. In each game there is a winner and a loser, the winner receives one point while the loser does not get any points. At the end of the tournament we list the number of points for each player, and compute the sum of the squares of these 10 numbers. Show that the number we got this way is at most 285.
5. Erin and Mikayla play the following game. They shuffle a regular deck of cards and Erin draws cards one by one until she finds the first black ace. Then Mikayla draws cards from the remaining deck one by one until she finds the remaining other black ace. The winner is the player who drew more cards, and there is a tie if they drew the same number of cards. Is this a fair game? If not, who has a bigger chance to win?
(A regular deck of cards has 52 cards, with two black aces. We assume that after shuffling the cards, all possible configurations are equally likely.)

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

Find old and current problems and information about the talent search at: <http://www.math.wisc.edu/talent>

Find an introduction to techniques for solving problems like these at: <https://goo.gl/pqq32m>

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Or Email To	talent@math.wisc.edu	Problem	Score
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Please check here if you would like to be emailed when new problem sets are available.