

**WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH  
 PROBLEM SET IV (2016-2017)**

**January 2017**

1. Find all the three-digit positive integers  $n$  such that if  $n$  is added to the number formed by writing the three digits of  $n$  in reverse order, the sum is 1372.
2. Let  $ABCD$  be a square. Find all possible positions of a point  $R$  on the segment  $CD$  such that there exist points  $P, Q$ , and  $S$  such that  $PQRS$  is a square, and  $Q$  is closer to line  $BC$  than any of the points  $P, R$ , or  $S$ .
3. We have an  $8 \times 8$  board consisting of 64 unit squares. We have thirty-two  $2 \times 1$  tiles which are built from two unit squares. Somebody placed 16 of these tiles on the board so that the tiles do not overlap, and each tile covers exactly two unit squares on the board. Show that we can place an additional (17th)  $2 \times 1$  tile on the board so that it covers exactly two unit squares without overlapping with any of the other tiles.

4. The numbers  $a_0, a_1, \dots, a_{2017}$  are not all zero, and they satisfy the following 2018 inequalities:

$$a_0 \geq 0, a_0 + a_1 \geq 0, a_0 + a_1 + a_2 \geq 0, \dots, a_0 + a_1 + \dots + a_{2017} \geq 0.$$

Show that there is no real number  $x$  greater than 1 that satisfies the equation

$$a_0x^{2017} + a_1x^{2016} + \dots + a_{2016}x + a_{2017} = 0.$$

5. Consider the collection of all the ordered pairs of sets  $(A, B)$  satisfying  $A \subseteq B \subseteq \{1, 2, 3, \dots, 2017\}$ . Find the average number of elements there are in the  $B$  sets of these ordered pairs. (Note that either  $A$  or  $B$  could be the empty set!)

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

Find old and current problems and other information about the talent search on our webpage:

<http://www.math.wisc.edu/talent>

<b>Return To</b>	<b>MATHEMATICS TALENT SEARCH</b> Dept. of Mathematics, 480 Lincoln Drive University of Wisconsin, Madison, WI 53706 talent@math.wisc.edu	<b>Deadline</b> February 6, 2017	
<b>Or Email To</b>		<b>Problem</b>	<b>Score</b>
<b>Please Fill In</b>	<b>PROBLEM SET IV</b>		
Name & Grade		1	
School & Town		2	
Home Address		3	
Town & Zip		4	
Email Address		5	
Teacher's Name			

Please check here if you would like to be emailed when new problem sets are available.