WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET III (2016-2017) **December 2016**

1. (a) Let a, b, c, and d be integers for which $ad \neq bc$. Show that it is always possible to write the fraction $\frac{1}{(ax+b)(cx+d)}$ in the form $\frac{r}{ax+b} + \frac{s}{cx+d}$ where r and s are fractions.

(b) Write the following sum as a common fraction:

$$\frac{1}{1 \cdot 4} + \frac{1}{4 \cdot 7} + \frac{1}{7 \cdot 10} + \cdots + \frac{1}{2998 \cdot 3001}.$$

2. In the trapezoid $ABCD$ the sides AB and CD are parallel and $BC = AB + CD$. Let F be the midpoint of AD. Find the angle BFC.

3. Find all pairs of integers m, n which solve the equation $\sqrt{n + \sqrt{n + \sqrt{n}}} = m$.

4. Find the maximal number of integers we can choose out of the numbers $1, 2, \ldots, 50$ so that their product is not divisible by 36. Make sure to prove that your choice is maximal!

5. We roll a fair die repeatedly and add up the numbers. We stop as soon as when we get to a number that is bigger than 1000. The possible numbers when we stop are 1001, 1002, ..., 1006. Which one of these numbers has the highest probability?

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

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