WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET V (2015-2016)   **February 2016**

1. There are 2016 points in the plane such that any three of the points are vertices of a triangle with an area of at most 1. Show that we can draw a rectangle of area 4 that contains all the points.

2. Bill writes down the numbers 1, 2, ..., 10 in every possible order. For each possible sequence he circles the numbers whose positions are equal to the number at that place and writes down with red the sum of the circled numbers, writing down 0 if there where none. (E.g. for the sequence $1, 3, 5, 4, 6, 7, 8, 9, 10, 2$ he would circle 1 and 4 and he would write down the number 5 in red.) What is the average of all the red numbers that Bill wrote down?

3. Show that if a, b, c are positive numbers with $a + b + c = 3$, then
$$\sqrt{2 + a^2} + \sqrt{2 + b^2} + \sqrt{2 + c^2} \geq 3\sqrt{3}.$$

4. Show that the number $(2016^2)!$ is divisible by $(2016!)^{2016}$. For an extra point show that it is also divisible by $(2016!)^{2017}$.

5. Two circles with radius 1 are tangent to each other, two circles with radius 4 are tangent to each other, and each circle with radius 1 is tangent to one of the circles with radius 4. All four of these circles are internally tangent to a larger circle as shown. Show that we can draw a small circle externally tangent to all four of the circles with radii 1 and 4. Find the radius of this small circle.
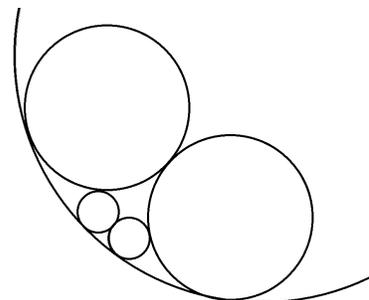
You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

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