

WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH
PROBLEM SET II (2015-2016) **November 2015**

1. Find every positive integer that is divisible by 8 where the sum of its digits is 7 and the product of its digits is 6.
2. We choose two different numbers randomly out of the numbers $1, 2, \dots, 50$. We assume that each possible pair of numbers is equally likely. What is the probability that the product of the two chosen numbers is divisible by 10?
3. The real numbers a, b, c , and d are each less than or equal to 12. The polynomial $p(x) = ax^3 + bx^2 + cx + d$ satisfies $p(2) = 2$, $p(4) = 4$, and $p(6) = 6$. What is the maximum possible value of $p(10)$?
4. Each of the diagonals of a convex quadrilateral $ABCD$ divide the quadrilateral into two equal area triangles. Show that $ABCD$ is a parallelogram.
5. A collection of line segments contained in a square of side length one inch is said to be *shading* if every straight line which intersects the square also intersects one of the line segments. Think of the line segments as painted walls in a house otherwise made of glass. For a shading set, a flashlight beam cannot pass through the house. E.g. if we take three of the sides of the square, or its two diagonals, then we have shading sets. If we take only two of its sides then we get a set which is not shading. A shading set need not consist of connected line segments. The lengths of the shading sets in our two examples are 3 and $2\sqrt{2} \approx 2.828$ inches. Find a shading set which has length less than 2.75 inches. You get an extra point if you can find one with total length less than 2.71 inches.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

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| 1 | |
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