

WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET I (2015-2016)

October 2015

1. Show that out of seven integers you can always find four so that the sum of these numbers is divisible by 4.
2. There is a circle that passes through the midpoints of all four sides of the quadrilateral $ABCD$. Show that the diagonals \overline{AC} and \overline{BD} are perpendicular.
3. How many three-digit positive integers satisfy the condition that any two consecutive digits differ by exactly one?
4. Let ℓ and m be two parallel lines one inch apart. We choose the line segment \overline{AB} on ℓ and the line segment \overline{CD} on m , each 2 inches long. Then for any position of the line segments we can move \overline{AB} so as to coincide with \overline{CD} in a way that the moving segment sweeps out a region of area exactly 2 square inches as it moves. One way to do this is by moving \overline{AB} parallel to itself within the parallelogram $ABCD$. Prove that if ℓ and m were 100 miles apart (instead of 1 inch), it would still be possible to move \overline{AB} to coincide with \overline{CD} while sweeping out a region of no more than 2 square inches.
5. Show that there is a finite list of distinct positive integers, each containing the digit 1, such that the sum of their reciprocals is more than 100.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

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