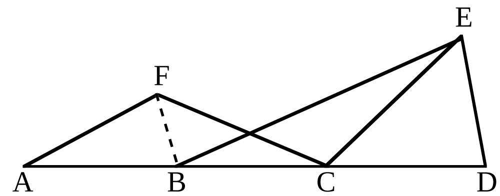
WISCONSIN MATHEMATICS SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET V (1999-2000) **February 2000**

1. Find all positive integers n so that $\frac{1}{n}$ is the repeating decimal $\frac{1}{n} = .abcabcabc\cdots = .\overline{abc}$ with a, b and c distinct digits between 0 and 9.

2. In the figure, $\overline{ABCD}$ is a straight line with $AB = BC = CD = 2$. Also $FA = DE = 2$, $BE = 4$, and $FC = CE$. Compute the distance FB.



3. Consider the sequence of integers $x_1 = 34,\ x_2 = 334,\ x_3 = 3334,\ \ldots,\ x_n = 33\cdots334,\ \ldots$ where the first n digits of x_n are 3s and the units digit is a 4. Compute the number of digits that are equal to 3 in the number $9(x_n)^3$.

4. Do there exist infinitely many triples (x, y, z) of real numbers which satisfy the system of equations

$$x^2 + xy + y^2 = y^2 + yz + z^2 = z^2 + zx + x^2 = 3$$

Justify your answer.

5. Suppose

$$1 = \frac{1}{a_1} + \frac{1}{a_2} + \cdots + \frac{1}{a_m}$$

where $a_1, a_2, \ldots, a_m$ are distinct positive integers. If the largest of the a_is is equal to $2p$ for some prime p, find the set $\{a_1, a_2, \ldots, a_m\}$.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on the problem set page. Remember that solutions usually require a proof or justification.

RETURN TO:	MATHEMATICS TALENT SEARCH	DEADLINE
	Dept. of Mathematics, 480 Lincoln Drive	March 13
	University of Wisconsin, Madison, WI 53706	2000

(Please Detach)

Last Name	First Name	Grade
School		Town
Home Address	Town	Zip Code

PROBLEM	SCORE
1	
2	
3	
4	
5	

PROBLEM SET V