1. Find all positive integers $n$ so that $\frac{1}{n}$ is the repeating decimal $\frac{1}{n} = .abcabcabc \cdots = .\overline{abc}$ with $a$, $b$ and $c$ distinct digits between 0 and 9.

2. In the figure, $ABCD$ is a straight line with $AB = BC = CD = 2$. Also $FA = DE = 2$, $BE = 4$, and $FC = CE$. Compute the distance $FB$.

3. Consider the sequence of integers $x_1 = 34$, $x_2 = 334$, $x_3 = 3334$, $\ldots$, $x_n = 33\cdots334$, $\ldots$ where the first $n$ digits of $x_n$ are 3s and the units digit is a 4. Compute the number of digits that are equal to 3 in the number $9(x_n)^3$.

4. Do there exist infinitely many triples $(x, y, z)$ of real numbers which satisfy the system of equations

$$x^2 + xy + y^2 = y^2 + yz + z^2 = z^2 + zx + x^2 = 3$$

Justify your answer.

5. Suppose

$$1 = \frac{1}{a_1} + \frac{1}{a_2} + \cdots + \frac{1}{a_m}$$

where $a_1, a_2, \ldots, a_m$ are distinct positive integers. If the largest of the $a_i$s is equal to $2p$ for some prime $p$, find the set $\{a_1, a_2, \ldots, a_m\}$.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on the problem set page. Remember that solutions usually require a proof or justification.

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