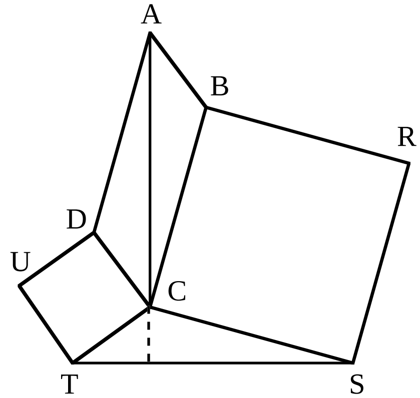
WISCONSIN MATHEMATICS SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET IV (1999-2000) **January 2000**

1. Let $\square$ be a binary operation defined on the integers, so that if x and y are any two integers, then $x \square y$ is some integer determined by x and y. Is it possible that $x \square (y \square z)$ is *never* equal to $(x \square y) \square z$ for any integers x, y, z? In other words, can a binary operation be "completely nonassociative"?



2. Squares $BRSC$ and $DCTU$ are constructed on two sides of the parallelogram $ABCD$, as shown, and line segments $\overline{AC}$ and $\overline{ST}$ are drawn. Prove that $AC = ST$ and that $\overline{AC}$ (extended) is perpendicular to $\overline{ST}$.

3. Suppose we are given a finite number of points on a circle, with each point labeled either 0 or 1. Then we can double the number of points by marking new points, one at each midpoint of the arc formed by two adjacent old points. Furthermore, if the two old points on either side of a new point have the same label, then the new point is labeled 0; otherwise it is labeled 1. Now start with two diametrically opposite points on the circle, one labeled 0 and the other 1, and repeatedly apply the doubling process as above. Prove that at every stage after the first, the total number of 0s is always equal to the number of 1s at the previous stage.

4. Let S be a set of 51 positive integers, each of which is at most 100. Show that S contains a number that is a multiple of some other number in S.

5. Let p be a prime number and suppose that $\dfrac{1}{p} = \dfrac{1}{a} + \dfrac{1}{b}$, where a and b are positive integers. Find all possibilities for p if one of a or b is a perfect square.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on the problem set page. Remember that solutions usually require a proof or justification.

RETURN TO:	MATHEMATICS TALENT SEARCH	DEADLINE
	Dept. of Mathematics, 480 Lincoln Drive	February 11
	University of Wisconsin, Madison, WI 53706	2000

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(Please Detach)

Last Name First Name Grade

School Town

Home Address Town Zip Code

PROBLEM	SCORE
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PROBLEM SET IV