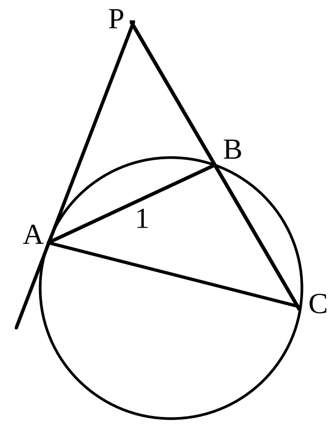
WISCONSIN MATHEMATICS SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET III (1999-2000) **DECEMBER 1999**

1. Let a, b and c be nonnegative real numbers. Prove that

$$(a+b+c)^3 \geq a^3+b^3+c^3+24abc\,.$$

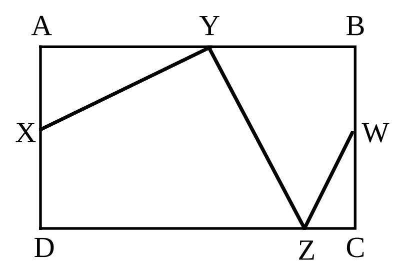
2. Triangle ABC is inscribed in a circle, and side $AB = 1$. The tangent to the circle at A meets the secant line through B and C at point P, and it happens that B is the midpoint of $\overline{PC}$. Compute the length of side AC.



3. (New Year's Problem) I wish to write the number 1 as a sum of reciprocals of 2000 different positive integers. Is this possible? Prove that your answer is correct.

4. Let p be an odd prime number and write $\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{p-1}$ as a fraction $\frac{a}{b}$ in reduced form, so that a and b are relatively prime positive integers. Prove that p divides a.

5. Suppose that the zigzag line $\overline{XYZW}$ has total length at most 2 units, and note that it is inscribed in the rectangle $ABCD$, as indicated. If $XD = WC$, show that the area of the rectangle $ABCD$ is at most 1 square unit.



You are invited to submit a solution even if you get just one problem. Please do not write your solutions on the page. Remember that solutions usually require a proof or justification.

RETURN TO: MATHEMATICS TALENT SEARCH
Dept. of Mathematics, 480 Lincoln Drive
University of Wisconsin, Madison, WI 53706

DEADLINE
January 7
2000

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(Please Detach)

Last Name First Name Grade

School Town

Home Address Town Zip Code

PROBLEM	**SCORE**
1	
2	
3	
4	
5	

PROBLEM SET III