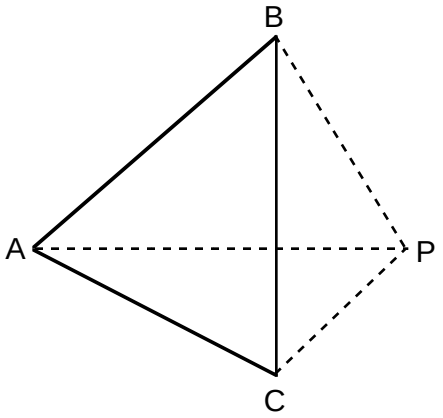
WISCONSIN MATHEMATICS SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET II (1999-2000) **NOVEMBER 1999**

1. Let S be a set of 51 different positive integers, each of which is at most 100. Show that there are two numbers in this set that differ by exactly 50.

2. Suppose that $\triangle ABC$ has acute angles, and that P is a point on the opposite side of line $\overline{BC}$ from A, as shown. If $PB^2 + AC^2 = PC^2 + AB^2$, prove that the line $\overline{AP}$ is perpendicular to $\overline{BC}$.



3. Find all positive integers N such that the quantity

$$s = \sqrt[3]{2+\sqrt{N}} + \sqrt[3]{2-\sqrt{N}}$$

is a positive integer. Prove that your answer is correct.

4. Show that a product of four consecutive (positive) integers cannot be a perfect square.

5. Given 20 points on a plane, 10 colored red and 10 blue; assume that no three are colinear. By a *connection* we mean a set of 10 line segments each one joining a different red point to a different blue one. How many connections are there? Now suppose we choose a connection with the property that the sum of the lengths of its 10 line segments is as small as possible. Prove that no two of its distinct line segments can intersect.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on the problem set page. Remember that solutions usually require a proof or justification.

RETURN TO:	MATHEMATICS TALENT SEARCH Dept. of Mathematics, 480 Lincoln Drive University of Wisconsin, Madison, WI 53706	DEADLINE December 1 1999

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(Please Detach)

Last Name First Name Grade

School Town

Home Address Town Zip Code

PROBLEM	SCORE
1	
2	
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4	
5	

PROBLEM SET II