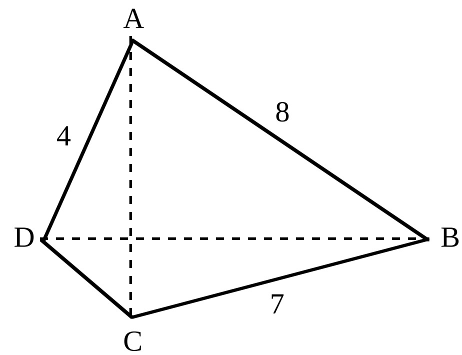
WISCONSIN MATHEMATICS SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET I (1999-2000) **OCTOBER 1999**



1. Let S be a set of 51 different positive integers, each of which is at most 100. Show that there are two members of S whose sum is 101.

2. In the figure, the diagonals AC and BD of the quadrilateral $ABCD$ are perpendicular. Given that $AB = 8$, $BC = 7$ and $DA = 4$, find CD, and prove that your answer is correct.

3. If n is any positive integer, write $\sqrt{n} = m + r$, where m is an integer and $0 \leq r < 1$. Of course, m and r depend upon n. For example, if $n = 20$, then $\sqrt{n} = 4.4721\ldots$, so $m = 4$ and $r = 0.4721\ldots$. Show that if n is a multiple of m, then either n is a square, n is 1 less than a square, or n is the product of two consecutive integers.

4. A number of high school students attend a dance. During the evening, each girl danced with at least one boy, but no boy danced with all the girls. Prove that there are two boys B_1 and B_2, and two girls G_1 and G_2, such that B_1 danced with G_1, B_2 danced with G_2, but B_1 did not dance with G_2, and B_2 did not dance with G_1.

5. Let n be a positive integer. Prove that the product of all odd numbers from 1 to $4n - 1$ (inclusive) can never exceed $(4n^2 - 1)^n$.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on the page. Remember that solutions usually require a proof or justification.

RETURN TO:	MATHEMATICS TALENT SEARCH	DEADLINE
	Dept. of Mathematics, 480 Lincoln Drive	November 1
	University of Wisconsin, Madison, WI 53706	1999

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(Please Detach)

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PROBLEM	SCORE
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PROBLEM SET I