

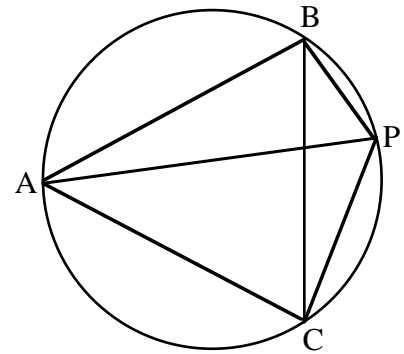
WISCONSIN MATHEMATICS SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET V (1998-99)

FEBRUARY 1999

1. For some real number a , the polynomial $x^4 - 10x^2 + a = 0$ has four real roots that happen to be equally spaced on the real number line. Find the four roots and prove that there are no other possibilities.

2. In the figure, equilateral $\triangle ABC$ is inscribed in a circle and a point P is chosen on the circle as shown, between vertices B and C . Show that $PA = PB + PC$.



3. Let \mathcal{T} be the set of all triples (a, b, c) of nonnegative integers. If $T = (a, b, c)$ is any member of \mathcal{T} , we write T^* to denote the new member of \mathcal{T} obtained by $T^* = (|b - c|, |c - a|, |a - b|)$. In other words, the operation $*$ replaces each entry in a triple by the difference of the other two entries. For example, we have $(2, 3, 4)^* = (1, 2, 1)$ and $(1, 2, 1)^* = (1, 0, 1)$. Prove that no matter what member of \mathcal{T} we start with, if we keep applying our operation $*$, we will eventually obtain a triple that contains the number 0.

4. Suppose that the prime number p can be written as a difference of two positive integer cubes. Show that when $4p$ is divided by 3 and the remainder is discarded, the result is the square of an odd integer.

5. Ten pennies are placed on a table with “heads” side up, and they are arranged in a circle. A “move” consists of simultaneously turning over three adjacent pennies. Is it possible to carry out a sequence of moves that results in the pennies all being “tails” side up, and if so, what is the smallest possible number of moves needed to accomplish this?

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on the problem set page. Remember that solutions usually require a proof or justification.

RETURN TO:

MATHEMATICS TALENT SEARCH
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DEADLINE
March 12
1999

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(Please Detach)

Last Name	First Name	Grade
School	Town	
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PROBLEM	SCORE
1	
2	
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PROBLEM SET V