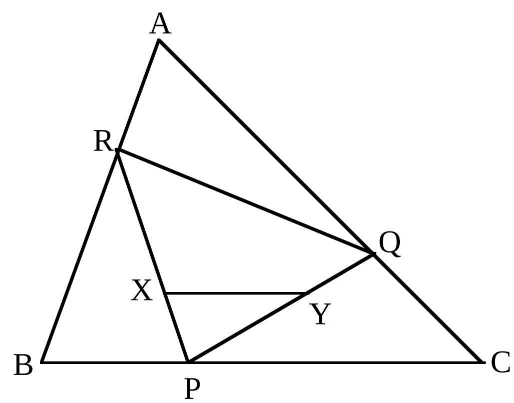
WISCONSIN MATHEMATICS SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET IV (1998-99) **JANUARY 1999**

1. Let a, b and c be real numbers with $a > 0$, and such that $a + b + c = 3$ and $abc = 1$. Find all possible values for a and prove that your answer is correct.

2. In the figure, points P, Q and R are chosen on the sides of $\triangle ABC$ so that P lies $1/3$ of the way from B to C, Q lies $1/3$ of the way from C to A, and R lies $1/3$ of the way from A to B. Then points X and Y on the sides of $\triangle PQR$ are chosen so that X lies $1/3$ of the way from P to R and Y lies $1/3$ of the way from Q to P. Prove that line $\overline{XY}$ is parallel to $\overline{BC}$.



3. Let m and n be positive integers and assume that the number $A = \dfrac{(m+n)^3}{n^2}$ is an odd integer. Find the smallest possible value that A can have, and find all pairs m and n which yield this value. Justify your answer.

4. Given a square with side of length 1, find the maximum number of points that can be placed in (or on the boundary of) the square such that no two of the points are closer than $1/2$ unit apart. Explain why your solution is correct.

5. In Problem Set I, we saw that a power of 2 can never be written as a sum of two or more consecutive positive integers. Every power of 3, however, can be written in this way. Find, with appropriate justification, the largest integer m such that 3^{11} can be written as a sum of exactly m consecutive positive integers.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on the problem set page. Remember that solutions usually require a proof or justification.

RETURN TO:	MATHEMATICS TALENT SEARCH	DEADLINE
	Dept. of Mathematics, 480 Lincoln Drive	February 12
	University of Wisconsin, Madison, WI 53706	1999

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(Please Detach)

Last Name First Name Grade

School Town

Home Address Town Zip Code

PROBLEM	SCORE
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PROBLEM SET IV