WISCONSIN MATHEMATICS SCIENCE & ENGINEERING TALENT SEARCH

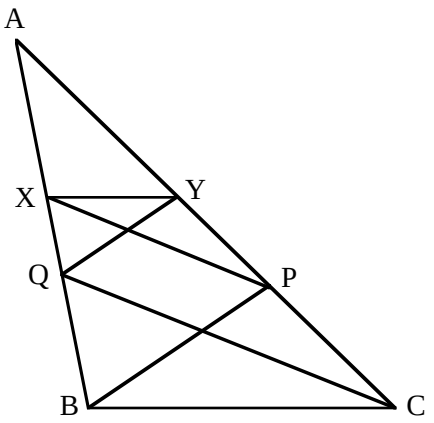
PROBLEM SET II (1998-99) **NOVEMBER 1998**

1. Find all prime numbers p for which it is possible to write

$$\frac{1}{p} = \frac{1}{a^2} + \frac{1}{b^2}$$

with positive integers a and b.

2. Points P and Q are selected on sides $\overline{AC}$ and $\overline{AB}$ of $\triangle ABC$, and line segments $\overline{BP}$ and $\overline{CQ}$ are drawn. Then $\overline{PX}$ and $\overline{QY}$ are drawn parallel to $\overline{CQ}$ and $\overline{BP}$, as shown. Prove that $\overline{XY}$ is parallel to $\overline{BC}$.



3. Your calculator will tell you that the quantity

$$z = \left(2\cdot\sqrt[3]{2} + 1 - \sqrt{12\cdot\sqrt[3]{2} - 15}\right)^3$$

is approximately an integer. Is z *exactly* an integer? Justify your answer.

4. If x, y and z are positive numbers, show that

$$8(x^3 + y^3 + z^3) \geq (x+y)^3 + (x+z)^3 + (y+z)^3 .$$

5. Let S be a subset of the set $\{1, 2, 3, \ldots, 99, 100\}$, and assume that S has at least ten members. Show that there are nonempty subsets X and Y of S such that X and Y have no members in common and the sum of the members of X is equal to the sum of the members of Y.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on the problem set page. Remember that solutions usually require a proof or justification.

RETURN TO: MATHEMATICS TALENT SEARCH
Dept. of Mathematics, 480 Lincoln Drive
University of Wisconsin, Madison, WI 53706

DEADLINE
December 2
1998

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(Please Detach)

Last Name First Name Grade

School Town

Home Address Town Zip Code

PROBLEM	SCORE
1	
2	
3	
4	
5	

PROBLEM SET II