

**WISCONSIN MATHEMATICS SCIENCE & ENGINEERING TALENT SEARCH**

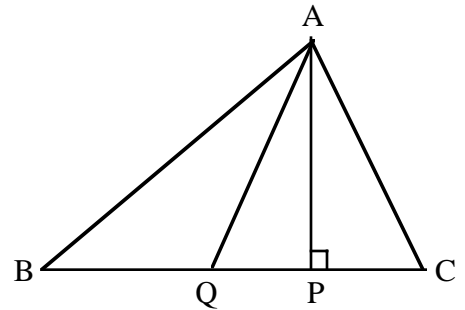
**PROBLEM SET I (1998-99)**

**OCTOBER 1998**

1. If  $x$  and  $y$  are any two real numbers, show that

$$(x^2 + y^2)^2 \geq xy(x + y)^2.$$

2. In the diagram,  $\overline{AP}$  is an altitude of  $\triangle ABC$ , and point  $Q$  is chosen on side  $\overline{BC}$  so that  $\angle BAQ = \angle CAP$ . Show that line  $\overline{AQ}$  goes through the circumcenter of  $\triangle ABC$ .



3. Observe that

$$2 \cdot 1^2 = 1^2 + 1$$

$$2 \cdot 2^2 = 3^2 - 1$$

$$2 \cdot 5^2 = 7^2 + 1$$

$$2 \cdot 12^2 = 17^2 - 1.$$

Show that there are actually infinitely many positive integer solutions to each of the equations  $2x^2 = y^2 + 1$  and  $2x^2 = y^2 - 1$ .

4. Suppose that every point in the plane is colored either red or blue. Prove that there exists an isosceles right triangle all of whose vertices have the same color.
5. Prove that no power of 2 can be written as a sum of two or more consecutive positive integers.

**You are invited to submit a solution even if you get just one problem. Please do not write your solutions on the problem set page. Remember that solutions usually require a proof or justification.**

RETURN TO:

MATHEMATICS TALENT SEARCH  
 Dept. of Mathematics, 480 Lincoln Drive  
 University of Wisconsin, Madison, WI 53706

DEADLINE  
 November 2  
 1998

(Please Detach)

Last Name	First Name	Grade
School		Town
Home Address	Town	Zip Code

PROBLEM	SCORE
1	
2	
3	
4	
5	

**PROBLEM SET I**